Towards a new model of speculative bubbles: nonparametric
test with an application to the Tunisian Stock Index

Abstract
Bubbles in asset prices have fascinated researchers in finance. Identify asset bubbles, by
circumstances, on the stock market has been a growing number of research theoretical and empirical.
On a theoretical level, it was assumed that the price dynamics reflect irrational behavior of economic
agents and, therefore, should be excluded from a deal with the truly rational economic agents
theory and behavioral theory attempted to explain why bubbles occur Allen and Gale [2000], Abreu &
Brunnermeier [2003], Scheinkman and Xiong [2003] and Men [2006]. This paper examines the
presence of rational speculative bubbles in the Tunisian equity market (BVMT) over a sample period
from January 2000 to December 2013 by means of a methodology based on a non-parametric duration
dependence test. The results show evidence of negative duration dependence in runs of positive
returns, a characteristic consistent with the presence of rational speculative bubbles. This paper
employs the generalized Weibull model of Mudholkar, Srivastava, and Kollia (1996) to examine the
nature of speculative bubbles in security prices. This model is sufficiently flexible to allow changes in
the direction of duration dependence

Keywords: Duration dependence, rational speculative bubbles, Weibull Model.
1. Introduction

Bubbles in asset prices have fascinated researchers in finance. Identify asset bubbles, by circumstances, on the stock market has been a growing number of research theoretical and empirical. On a theoretical level, it was assumed that the price dynamics reflect irrational behavior of economic agents and, therefore, should be excluded from a deal with the truly rational economic agents Burmeister [1980], Cass and Shell [1983], Tirole [1985] and Diba & Grossman [1988]. Rational theory and behavioral theory attempted to explain why bubbles occur Allen and Gale [2000], Abreu & Brunnermeier [2003], Scheinkman and Xiong [2003] and Men [2006]. In fact, Gürkaynak [2008] studied the phenomenon of rational price bubbles by conducting a range of econometric tests and found that the empirical results are mixed: are research who find evidence of bubbles, there are other which correspond to the data without allowing both to detect the presence of a bubble. Recently, many researchers have turned to the paradigm of behavioral finance Men & al [2004] and Shiller [2003]. However, Brav & Heaton [2002] find it difficult to distinguish between "......the behavioral theories based on the irrationality of investors and theories of rational structural uncertainty based on incomplete information attached to the structure of the environment of the economy .... ".

From empirical perspective, on the other hand, the main interest was simply to verify that the price series in question is best explained by assuming the presence of speculative bubbles otherwise. Accordingly, various statistical approaches have been primarily concerned with the tests in order to highlight the existence and duration of bubbles. They are based, in one way or another on the prima facie-abnormalities caused by the divergence or non-stationary prices Flood & Garber [1980], Blanchard & Watson [1982] and West [1987]. Solis [2006] examined the evolution of the persistence of price-dividend stock index S & P500 by screening breaks the integration of zero I (0) in the order of integration I (1) in time series, while Phillips & al [2011]. Phillips & Yu [2011] used before the period of recursive regression period to assess the evidence behavior unit root against a slightly explosive alternative. Gutierrez [2011] proposes a methodology based on the technique 'bootstrap' to calculate the probability distribution of the finite asymptotic tests proposed by Phillips et al sample. [2011]. Homm & Breigung [2012] took the procedure Phillips & al [2011] than other test procedures of Bhargava [1986] Busetti & Taylor [2004], Kim [2000] and Kim & t al [2002] detects presence of rational bubbles. Their simulations show that the estimator from the Chow test is more reliable and relevant with finite samples. Cerqueti & Constantini [2011] analyzed international data covering 18 OECD countries; and with the integration and co-integration tests; proved the existence of bubbles in the period 1992-2010. Al-Anaswah & Wilfling [2011] have applied state-space models with Markov switching to simulate international data.

Needless to say that a bubble is characterized by a divergent sequence of a price at its early stage of development is followed by a sudden crash in time. When a bubble inflates, the evolution over time of the price divergence and when it crashes, there is a discrete jump in the time series data. This function, although important in the definition of bubbles, has hardly been analyzed official statistics. McQueen & Thorley [1994] used the statistical theory of duration dependence to model speculative bubbles and even more, to examine the monthly returns of NYSE portfolio to put into evidence the phenomenon.

Rational speculative bubbles refer to a state in which the price of an asset is explained by factors other than the market valuation and the actual value of DeMarzo & al [2007] active. Assert that prices can be driven up when some uninformed investors boost prices to deviate from fundamentals. Other investors buy assets with the assumption that the price will rise, and they can sell it before the price tends to fall. This pushes the price away from its fundamental value. The result is a speculative bubble in the short term it deflates when the price returns to its fundamental value [Kirman & Teyssiere (2005); Dass & al (2007)]. Therefore, in this study, we examine whether the Tunisian market price TUNINDEX was characterized by rational bubbles during the period January 1992 to May 2013 This issue remains at the heart of market practice as far as the detection of bubbles rational speculation can be very useful for investors and policy decisions. Point saver view, the fact of knowing the size of the bubble allows him to help him recognize the signs of a possible collapse of possibility of financial asset prices (Brooks & Katsaris, 2005). For policy makers, be aware of the possibility and the size of the bubbles provides more guidance on how to protect the financial market by reducing the consequences of such an attack through more efficient handling of regulatory policies and control. Hence the main interest is to study the interaction between price levels and dividends using a cointegration test. This test is based on the idea that if a rational bubble exists and if dividends are stationary in first differences, the series of stock prices (in first differences) should be stationary (ie of stock prices are more explosive than dividends). Conversely, the absence of a co-integration relationship between prices and fundamental values indicates the existence of a rational bubble. This test has been used in several previous studies, including McQueen and Thorley [1994], Chan & al [1998] Lavin & Zorn [2001], Harman & Zuelke [2004] and Jirasakuldech & al [2008]. However, Evans [1991], Charemza & Deadman [1995], Brooks & Katsaris [2003], Sarno & Taylor [2003], Kanas [2003, 2005], Chang & al [2007] and Waters [2007]. These authors have demonstrated that traditional tests cointegration is unable to detect an important class of rational bubbles, namely the periodic collapse of bubbles due to these types of non-linear characteristics of bubble. Given the above concerns with the methodology of cointegration, we use in this study a method that does not require the identification of fundamental factors, namely, the test of dependence of duration. This method examines the stock returns of the presence of negative duration dependence (or decrease in risk) in yields positive runs, a unique feature of rational expectations bubbles (McQueen and Thorley, 1994). This approach is based on the argument that if the stock price will experience episodes of rational bubbles, stock returns exhibit a negative duration dependence where the probability of a negative abnormal return in the current period is a decreasing function length set of positive abnormal returns. The method of
time-dependence overcomes criticism addressed to the cointegration technique that is a joint test of the null hypothesis of "no bubble" and "no modeling errors." The added benefit of test duration dependence is that it does not require the normality of the time series Jirasakuldech & al [2008].

2. Hypotheses and Methodology:

The theoretical model developed by rational bubbles McQueeen and Thorley [1994] suggests that the process of bubble led to changes explosive price. According to this model, rational speculative bubbles may exist if positive returns have a negative duration dependence, i.e., the conditional probability that ends, given its length, is a decreasing function length of runs. In other words, if stock prices are influenced by rational expectations bubbles, price changes exhibit negative duration dependence - or a rate of decreasing risk - where there is a negative relationship between the probability of a term positive will end and the length of the positive race.

Following the work of Chan & al (1998) and Jirasakuldech & al (2008), a test duration dependence was applied by examining the rate of probability \( \hat{h}_i = N_i / (M_i + N_i) \) can be estimated from its function of maximizing log-likelihood function risk.

\[
L[\theta / S_i] = \sum_{i=1}^{\infty} N_i Lnh_i + M_i (1 - h_i) + Q_i (1 - h_i)
\]  

Where \( \theta \) is a parameter vector, \( S_i \) is the set of \( T \) observations on the length of random data runs \( i \), \( N_i \) is the number of complete cycles of runs of length \( i \) and \( M_i \) and \( Q_i \) are respectively the complete and partial number of runs of length greater than \( i \). To test the dependence of the duration, a shape function must be selected from the hazard function. This study used the Weibull hazard model based on risk. The risk Weibull function is defined as follows:

\[
h(t) = \alpha (\beta + 1) t^\beta
\]  

Or logarithmic term:

\[
Lnh(t) = Ln[\alpha (\beta + 1)] + \beta Lnt
\]  

Where \( \alpha > 0 \), \( \beta > -1 \) and \( t > 0 \).

The equation illustrates the fundamental assumption of the Weibull specification. There is a linear relationship between the logarithm of the hazard function and the logarithm of the duration. The elasticity of the duration is defined as the derivative of \( \ln[h(t)] \) versus \( \ln(t) \) and represented graphically as the slope of the function log-risk. Under the null hypothesis of no bubble, the hazard rate must be constant [ \( H_0 : \beta = 0 \) ]. The alternative hypothesis of a
bubble is a decreasing rate risk or negative duration dependence in runs of positive returns. A decreasing rate risk means that the probability of the positive end of a term should decrease with the length of runs \( H_0 : \beta < 0 \).

3. Application on the Tunisian stock market:

In this section, we apply our model to the Tunisian stock market. The monthly time series data we have chosen are the general index of the Tunis Stock Exchange TUNINDEX, interest and gross domestic product GDP. The theoretical basis of the share price is equal to the present value of expected future dividends if the stock market is efficient. However, such measures in practice are hardly computable since the expected values of future variables are unobservable. Furthermore, it should be noted that the performance of equity markets in the United States and other countries has long been questioned, based on a number of empirical findings. As part of this research, and for simplicity and in order to maintain consistency in the Tunisian context, we adopt as comparative two factors reflecting the fundamental studies: nominal GDP and the interest rate. So in order to eliminate movement from the changes due to market fundamentals, we regress the monthly original stock price indices in logarithms on the level of interest rates and the logarithm of nominal GDP as follows:

\[
\ln(\text{Index}_t) = \text{constant} + a \ln(\text{Interest Rate}_t) + b \ln(\text{GDP}_t)
\]

Case of the Tunisian stock market:

The sampling period is TUNINDEX from January 2000 to December 2013 estimated results for the basic equation is:

\[
\begin{align*}
\ln(\text{Index}_t) &= 15.729 - 1.470 \times \ln(\text{Interest Rate}_t) - 0.104 \ln(\text{GDP}_t), \\
(1.307) & (5.774) (1.683)
\end{align*}
\]

Where \( \text{Index}_t \), \( \text{Interest Rate}_t \), and \( \text{GDP}_t \) represent the stock index TUNINDEX, the interest rate and the nominal GDP rates and figures in parentheses are t-statistics. We can say that GDP is the primary determinant of financial activity in the stock market. The interest rate also affects the stock market. However, the marginal effect is very small compared to that detected for the GDP by the financial literature that has been developed in developed financial markets. A relatively high coefficient of determination \( R^2 \) indicates that TUNINDEX reflect the fundamentals of the economy. However, the effect of nominal GDP is considered quite low resulting in the small value \( R^2 \). This indicates that the definition of basic assumptions for developed economies is violated and there may be other factors that influence the movement of the Tunisian stock market. Similarly, we can show that the price dynamics of financial assets; common feature for all equity markets; has a tendency to keep well clear of the fundamental where the existence of speculative bubbles remain responsible for these erratic movements.

Data and empirical results:

We study in this paper the monthly behavior of the Tunisian general market index (TUNINDEX) over the period 2000-2013. The choice of monthly data is justified for the following reasons: the previous literature suggests that monthly to weekly data are favored because they reduce the signal to noise ratio that can cause runs associated with speculative bubbles (McQueen and Thorley [1994] and Chan et al [1998]). As part of testing for the presence of bubbles of rational expectations, the noise introduced by changes...
Towards a new model of speculative bubbles: nonparametric test with…

in fundamental price may complicate further the detection of bubbles in financial markets. Second, the theory of bubble gives no indication of the usual duration of a bubble, although the empirical literature implies that bubbles can accumulate over several months. The duration dependence test to detect the presence of rational bubbles is conducted to answer these questions. To better understand the behavior of the stock TUNINDEX, Figure (1) illustrates the evolution of monthly returns.

We present in Table [1] some evidence of the behavior of the stock market based on the values of the mean, standard deviation, minimum, maximum, the skewness and kurtosis of the series of returns examined. Statistic of the normality of the series highlighted is also presented (statistics Jarque Bera test & 1980). In fact, TUNINDEX is characterized by an average yield of 0.163% with a spread of 0.4458%. The presence of bubbles in the Tunisian market index can be checked by strong evidence of asymmetry, kurtosis and non-normality of the distribution of returns. Indeed, TUNINDEX is marked by a significant asymmetry and a leptokurtosis [Statistics kurtosis> 3], that is to say, the study presents series "fat tails" in relation to a normal distribution, as indicated by the the important kurtosis.

The Jarque-Bera as shown in Table [1] rejects the null hypothesis of normality of the statistical distribution to a significance level of 1%. The tail of the distribution of returns of stock market implies that price changes may differ quite large scale: an indication for the presence of bubbles on the Tunisian stock market. The model of rational bubbles also implies positive autocorrelation of returns during the period in which the bubble grows. Using the autocorrelation Q (6) and Q (12) statistics reported in Table [1], we reject the null hypothesis of no autocorrelation in two shifts. Overall, the consistent evidence of kurtosis and autocorrelation strengthens the possibility that the bubbles could be part of erratic movements of the Tunisian stock market index. However, the evidence supporting the existence of bubbles is not conclusive because other factors could generate the return series. For example, the two criteria of asymmetry and kurtosis of the series of returns can be the resultant of changes in economic fundamentals or through the application of economic policy rather than the existence of speculative bubbles [Westhoff, 2003].
Table [1]: Descriptive Statistics of Monthly Returns.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>St. Dev</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>J.B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Descriptive statistics of monthly returns.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.1630</td>
<td>-1.3765</td>
<td>0.018053</td>
<td>0.4458</td>
<td>-0.00835**</td>
<td>4.060616*</td>
<td>12.04885*</td>
</tr>
<tr>
<td>Panel B: Auto-correlation of monthly returns.</td>
<td>ρ₁</td>
<td>ρ₂</td>
<td>ρ₃</td>
<td>ρ₄</td>
<td>ρ₅</td>
<td>ρ₆</td>
<td>φ₆(6)</td>
</tr>
<tr>
<td></td>
<td>0.409</td>
<td>0.250</td>
<td>0.114</td>
<td>0.148</td>
<td>0.010</td>
<td>-0.031</td>
<td>69.174*</td>
</tr>
</tbody>
</table>

Table [2]: Application of test runs of rational speculative bubbles.

<table>
<thead>
<tr>
<th></th>
<th>MLE</th>
<th>S.E.</th>
<th>P. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A : Runs Positifs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α</td>
<td>0.4215*</td>
<td>0.0383</td>
<td>0.000</td>
</tr>
<tr>
<td>β</td>
<td>-0.0924*</td>
<td>0.0296</td>
<td>0.001</td>
</tr>
<tr>
<td>Panel B : Runs Négatifs</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>α</td>
<td>-0.0876</td>
<td>0.0707</td>
<td>0.140</td>
</tr>
<tr>
<td>β</td>
<td>0.3175</td>
<td>0.2251</td>
<td>0.187</td>
</tr>
</tbody>
</table>

Notes: * and ** indicate significance respectively at the 1% and 5%.

Table [2] shows the estimated test duration dependence on the positive and negative runs of monthly returns. We present the ML [Maximum Likelihood] model parameter Weibull estimates and standard errors and P-value of the corresponding t statistic. During the sample period, we observe 46 runs of positive returns and 39 negative runs for a total of 85 runs obtained. However; as we noted earlier; the null hypothesis of non-existence of a speculative bubble implies a constant rate of \( \beta = 0 \) risk. The alternative hypothesis of the presence of speculative bubbles implies a risk function of negative slope \( \beta < 0 \) for positive runs. In the light of Table [2], we also find that the TUNINDEX has a significantly negative coefficient \( \beta \) of -0.0924 with a significance level of 1%. While the nature of speculative bubbles, we cannot extrapolate the existence of rational speculative bubbles in the negative returns of the series of TUNINDEX. As the table shows, the null hypothesis of non-dependence of runs or the rate constant risk \( H_0 : \beta = 0 \) cannot be rejected at traditional significance.

To follow the evolution of the Tunisian stock index over a 23 year period and test this study has the advantage if victim rational speculative accidents.

This study examines the existence of rational speculative bubbles in the TUNINDEX Tunisian stock index. It covers monthly data from January 2000 to December 2013 (Which she was conducted on all monthly observations covering the period January 2000 until December 2013 tests on yields are conducted using most questionable models of speculative bubbles namely technical dependency runs. implications as a result, we note that the returns of the Tunisian stock index are characterized by positive autocorrelation, a negative asymmetry and leptokurtosis which justifies although the presence of rational speculative bubbles. nonparametric test runs more
corroborates other evidence of the presence of rational bubbles in our case. We postulate that the dynamics of monthly returns of the TUNINDEX is exposed to respect to the negative duration dependence in runs of positive returns, a feature consistent with the presence of rational bubbles. Results of this study provide several implications for investors and especially policy makers. The existence of a rational bubble can lead to a misallocation of resources and have devastating social and economic fluctuations. Therefore, policy makers and market makers must be aware of the harmful effects of irrational behavior and empowered to regulate and intervene to correct negative externalities induced.

4. Conclusion:
During the last two decades, financial literature has amassed a substantial number of stock anomalies which remains at odds with the hypothesis of efficient capital markets. However, some financial market phenomena such as mergers / acquisitions characterized by a certain fragility in addition to the consensus of the various financial actors can lead to the formation of bubbles and market crashes that these decisions remain flawed and not justified from fundamentals. However, the study of mimetic phenomena remains very important as it allows us to elucidate the detection of certain abnormalities detected on the stock exchanges that the mimicry; is one of our most basic instincts; and the strong belief that financial participants are influenced; first order; by the decisions of peers. In this respect, facing a series of standard questions which focuses on the explanation of the violation of the principle of random walk in stock prices, creating bubbles and excessive volatility of returns.

The results identified in the light of tests developed in the framework of research allow us to reject the hypothesis of absence of speculative bubbles on the Tunisian stock market, which we call to provide explanations; which bubbles suggest being as an answer; Dynamics of changes in financial asset prices. Still, the results developed in our research context should be reconsidered as the first recommendations that require better refine advantage through further testing. Indeed, we have suggested a new model for speculative bubbles in elucidating the presence of the latter on the Tunisian stock market through the application of non-parametric test runs. Consistent with the presence of rational speculative bubbles, the dynamics of returns to TUNINDEX are characterized by positive autocorrelation, a negative asymmetry and leptokurtosis. The nonparametric test runs further corroborates other evidence of the presence of rational bubbles in our case. We distinguish in this connection that the dynamics of monthly returns of the TUNINDEX is exposed towards the negative duration dependence in runs of positive returns, a feature consistent with the presence of rational bubbles.

Bibliographies:
• Vol. 55, p. 319-346.
• Calverley, John (2004), Bubbles and How to Survive Them, Nicholas Brealey Publishing, Finland.

Vol.2 / No.6 / Spring & Summer 2012


